Operators

Ậf(X) = g(X) : General definition of operator

It is defined as , an operator is a rule that transforms a given function f into another function. The simple definition is the operator is a symbol that tells you to do something to what follows the symbol. We indicate an operator by capital letters with tent also called a hat over it e.g. Û,Ĉ,Ĵ,Ô,Ŝ,Ŵ

Thus:

√ is an operator tilling you to take the squar root

d/dx ……………………….to differentiate w.r.t.x

x …………………………….to multiply by x

By using the definition equation Ậf(X) = g(X) we get

OperatorẬ	Function f	Ậf(X)
d/dx	f	f (X)
3	f	3f
Cos( )	x	cosx
√	x	(X)↑1/2

There are two very important operators:

$\nabla = d/dx + d/dy + d/dz$

$\nabla 2 = d2/dx2 + d2/dy2 + d2/dz2$

When two or more operators act on the same function, the by convention we take always begin with the operator on the right and work toward the left, because operators very often do not commute, i.e. ÔÛ ≠ ÛÔ

$$\hat{p} = \frac{\partial}{dx} \quad , \quad \hat{\wp} = x \quad , \quad f(x) = 3x^2$$

show That $\hat{p}$ and $\hat{\wp}$ do not commute

$$\hat{p}\hat{\wp} f(x) = \hat{p}\hat{\wp}\, 3x^2$$
$$= \hat{p}\, 3x^3 = \boxed{9x^2}$$

$$\hat{\wp}\hat{p} f(x) = \hat{\wp}\hat{p}\, 3x^2$$
$$= \hat{\wp}\, 6x = \boxed{6x^2}$$

$$\hat{p}\hat{\wp} \neq \hat{\wp}\hat{p}$$

$\therefore$ $\hat{p}$ and $\hat{\wp}$ are not commutative.

$$\hat{p} = \frac{\partial}{dx} \quad , \quad \hat{\wp} = \text{a Constant} \quad f(x) = 3x^2$$

Show That $\hat{p}$ and $\hat{\wp}$ are commutative

$$\hat{p}\hat{\wp} f(x) = \hat{p}\hat{\wp}\, 3x^2 = \hat{p}\, a3x^2 = \boxed{a6x}$$

$$\hat{\wp}\hat{p} f(x) = \hat{\wp}\hat{p}\, 3x^2 = \hat{\wp}\, 6x = \boxed{a6x}$$

$$\hat{p}\hat{\wp} = \hat{\wp}\hat{p}$$

$f(x) = x^2$, $\hat{P} = \sqrt{}$ and $\hat{G} = 2x^0$ (2)

$$\hat{P}\hat{G} f(x) = \hat{P}\, 2x^2$$
$$= \sqrt{2x^2} = x\sqrt{2}$$

$$\hat{G}\hat{P} f(x) = \hat{G}\sqrt{x^2}$$
$$= \hat{G} x = 2x$$

$\therefore \hat{P}\hat{G} f(x) \neq \hat{G}\hat{P} f(x)$

$\therefore \hat{P}$ and $\hat{G}$ do not commute or are not commutative.

An operator is said to be linear if :-

$$\hat{P}(f+g) = \hat{P}f + \hat{P}g$$

or

$\hat{P}af = a\hat{P}f$ where a is a constant

examples

Because $\sqrt{4+16} \neq \sqrt{4} + \sqrt{16}$ $f=4, g=16$

$\sqrt{20} \neq 2+4$

$\therefore \sqrt{}$ is nonlinear operator.

$\hat{p}\, a f = a \hat{p} f$ assume $f(x) = 5x^3$

$\hat{p}\, a\, 5x^3 = \frac{\partial}{dx}(5ax^3)$ where a is a con

$= \boxed{15ax^2}$

$a \hat{p} f(x) = a \frac{\partial}{dx}(5x^3) = \boxed{a\, 15x^2}$

$\therefore \frac{\partial}{dx}$ is a linear operator

Some linear operators $x, x^2, \frac{\partial}{dx}, \frac{\partial^2}{dx^2}$

Some non-linear operators $\cos, \sqrt{\ }, [\ ]^2$